FORMULATION AND SOLUTION OF THE DEFLECTION AND SECTION ROTATION EQUATION FOR THE WARP BEAM ON WEAVING LOOMS

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Abstract. The commodity roller is one of the key components of a weaving loom, ensuring uniform fabric tension during the winding process. Its proper functioning affects the quality of the product, the stability of the loom's operation, and the efficiency of the production process. With increasing demands for precision, reliability, and durability in textile industry equipment, there is a growing need for a detailed analysis of the mechanical behavior of rollers, including their deflection, rotation, and deformation. The development of mathematical models enables improvements in equipment performance, reduces the risk of failures, and extends the lifespan of components. This research is relevant for enhancing the competitiveness of modern weaving technologies and meeting the demand for high-quality textile products. This study investigates the process of winding fabric onto the commodity roller of weaving looms, with a focus on the formulation and solution of deflection and section rotation equations for the warp beam. The mechanical interactions between fabric tension, roller geometry, and beam deformation are analyzed. To determine the deflection and rotation angle of the beam, a calculation model is developed. The findings of the research contribute to enhancing the efficiency, durability, and performance of weaving looms by improving the design.

Keywords: *weaving loom, fabric take-up mechanism, commodity roller, warp beam, beam deflection.*

1. Introduction.

The textile industry is one of the oldest and most significant sectors of the global economy, with a rich history that dates back thousands of years. It encompasses the production of fibers, yarn, fabrics, and finished goods, including clothing, home textiles, and industrial textiles. As a fundamental component of the global supply chain, the textile industry plays a crucial role in the manufacturing process and contributes significantly to economic development and employment worldwide.

In modern times, the textile industry has evolved through technological advancements and innovations. The introduction of automation, the use of synthetic fibers, and the development of advanced weaving and knitting techniques have all transformed production processes, enhancing both speed and efficiency. Additionally, the industry's focus on sustainability and eco-friendly practices has grown, with an emphasis on reducing waste, conserving resources, and utilizing renewable materials.

Weaving looms are essential machines in the textile industry, playing a pivotal role in the production of woven fabrics. These machines interlace two sets of yarn—warp and weft—to create textiles for various applications, from clothing to industrial products. The evolution of weaving looms has been a major factor in the development and expansion of the textile industry, enabling higher production speeds, improved fabric quality, and greater efficiency.

Weaving looms operate on the principle of controlling the movement of yarns through a series of mechanical actions. The warp yarns are held taut on the loom, while the weft yarns are inserted through the warp by a shuttle, rapier, or air jet, depending on the type of loom. The process of weaving involves several key actions, including shedding (raising and lowering the warp threads), picking (inserting the weft thread), and beating-up (pushing the weft into place). These operations are precisely controlled to produce a wide variety of fabrics, from simple plain weaves to more complex patterns.

Modern weaving looms have evolved to include advanced features, such as automatic shuttle change, computerized control systems, and improved tension regulation, which enhance fabric quality, reduce waste, and increase production efficiency. In addition to conventional looms, there are specialized machines designed for specific textile products, such as jacquard looms for intricate patterns or air-jet looms for high-speed production.

As a critical part of the textile manufacturing process, weaving looms continue to drive innovation in the industry. They are at the forefront of meeting the growing demand for diverse and high-performance fabrics while adapting to the challenges of sustainability, automation, and cost efficiency.

The performance and efficiency of weaving looms are significantly influenced by the behavior of key components, such as the warp beam (commodity roller). The warp beam plays a critical role in maintaining the correct fabric tension during the weaving process. Understanding the mechanical interactions that determine the deformation of the warp beam is essential for optimizing loom operation and ensuring high-quality fabric production.

This study focuses on the formulation and solution of the deflection and rotation equations for the warp beam, an important aspect of modeling the forces and motions that affect the beam during its operation. Accurate prediction of deflection and rotation is necessary to prevent mechanical damage, reduce wear, and improve the overall performance of weaving looms.

The analysis involves developing a calculation model for determining the deflection and rotation angle of the warp beam. This model takes into account material properties, geometric parameters, and external forces acting on the beam. It will provide a deeper understanding of the mechanical behavior of the warp beam during operation and help optimize the design of weaving looms, improving both efficiency and durability. The results of this research may provide valuable insights for engineers and manufacturers in the textile industry, contributing to the advancement of weaving technologies.

When external forces act in one of the principal planes of inertia of the roller, the axis of the roller bends in the same plane, causing a flat bending. The displacement of the center of gravity of the section in a direction perpendicular to the axis of the roller is called the deflection of the roller in this section, or the deflection of this section of the roller. The deflection of the roller will be denoted by the symbol y. When the roller deforms, the section remains flat and rotates relative to its previous position. The angle θ, by which the section rotates relative to its initial position, is called the angle of rotation of the section [1].

To determine the deformation of the roller, it is necessary to be able to calculate the deflection y and the angle of rotation θ for each section. Both are functions of x, the distance of the section from the origin of coordinates, and between y and θ, for which the section of the roller has a defined relationship [2, p. 45].

Thus, the problem of studying the deformation of the roller is reduced to obtaining the equation of the bent axis $y = f(x)$. Knowing this, it is possible to calculate the angle of rotation for any section of the roller by differentiation.

2. The formulation of the differential equation of the bent axis of the roller.

In order to obtain the deflection y as a function of x (the length of the roller), it is necessary to establish the relationship between the deformation of the roller and the external forces that bend it, as well as its dimensions and material properties [3]. For the calculation of deflection and angle of rotation of the roller, we use the computational diagram shown in Figure 1.

The computational diagram represents a shaft of constant value, which is positioned on two supports, in two planes with different distributing and constituent forces [4].

The differential equation of the bent axis of the roller in both planes is as follows:

$$\pm EJ \frac{d^2y}{dx^2} = M(x) \tag{1}$$

The sign of the bending moment will be determined based on the known rule [5].

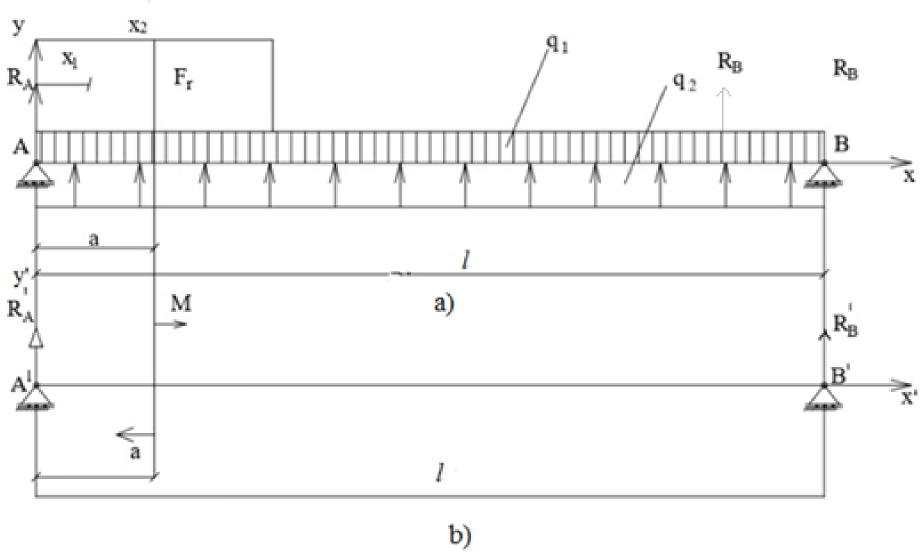

Figure 1. Computational diagram for determining the deflection and rotation of the roller section under the first loading method and the distribution of the gear transmission in the left support

To obtain the equation of the deflection $y = f(x)$ from the differential equation of the bent axis, it is necessary to integrate equation (1). The expression $M(x)$ is a function of x. Therefore, integrating equation (1) twice will yield:

$$\pm EJ \frac{dy}{dx} = \int M(x)dx + C \tag{2}$$

$$E\,J\,y = \int dx \int M(x)dx + Cx + D \tag{3}$$

Thus, we will obtain the equations for the angle of rotation and the bending of the roller in the following form:

$$\theta = \frac{dy}{dx} = \frac{1}{E\,J}\left[\int M(x)dx + C\right] \tag{4}$$

$$y = \frac{1}{E\,J}\left[\int dx \int M(x)dx + Cx + D\right] \tag{5}$$

Here, C and D are the constants of integration. The obtained relationship will be applied to determine the angle of rotation θ and the deflection of the roller section .

The origin of coordinates is taken at point A, with the y-axis directed upwards and the x-axis directed to the right along the length AB of the roller.

In this problem, to formulate the expression $M(x)$, it is necessary to determine the support reactions.

According to Figure 1a, it is possible to write:

$$\Sigma M_A = R_B\, l + q_2 \frac{l^2}{2} + F_r\, a - q_1 \frac{l^2}{2} = 0 \tag{6}$$

$$\Sigma M_B = R_A\, l + q_2 \frac{l^2}{2} + F_r\,(l-a) - q_1 \frac{l^2}{2} = 0 \tag{7}$$

And

$$R_A = (q_1 - q_2)\frac{l}{2} - F_r \frac{(l-a)}{l} \tag{8}$$

$$R_B = (q_1 - q_2)\frac{l}{2} - F_r \frac{a}{l} \tag{9}$$

We sequentially compute:

$$EJ \frac{d^2y}{dx^2} = M(x) \tag{10}$$

In the computational diagram of the roller, for determining the moment *M(x)*, there are two sections. Each section has two constants of integration: C_1, D_1 for the first section, and C_2, D_2 for the second section. Therefore, to solve the problem, certain rules must be introduced to reduce the number of constants to two, independent of the number of loading sections [6].

The expression for the bending moment in the second section must be formulated such that the terms in the equations EJy'', EJy' and EJy coincide with the corresponding terms in the equations of the first section. This is ensured by integrating the bracket (x-a), which represents the arm of the load missing in the first section, with respect to $d(x-a)$, or as it is said, by not expanding the brackets.

Here, x – is the abscissa of the current section of the considered segment;

a – is the abscissa of the beginning of this segment.

For I segment $0<x_1<a$

$$M(x_1) = R_A x_1 + q_2 \frac{x_1^2}{2} - q_1 \frac{x_1^2}{2} \tag{11}$$

$$EJ \frac{d^2y}{dx^2} = R_A x_1 + (q_2 - q_1) \frac{x_1^2}{2} \tag{12}$$

$$EJ \frac{dy}{dx} = R_A x_1^2/2 + (q_2 - q_1) \frac{x_1^3}{6} + C_1 \tag{13}$$

$$E J y_1 = R_A x_1^3/6 + (q_2 - q_1) \frac{x_1^4}{24} + C_1 x_1 + D_1 \tag{14}$$

For II segment $0<x_2<l$

$$M(x_2) = R_A x_2 + (q_2 - q_1) \frac{x_2^2}{2} + F_r (x_2 - a) \tag{15}$$

$$EJ \frac{d^2y}{dx^2} = R_A x_2 + (q_2 - q_1) \frac{x_2^2}{2} + F_r (x_2 - a) \tag{16}$$

$$EJ \frac{dy}{dx} = R_A x_2^2/2 + (q_2 - q_1) \frac{x_2^3}{6} + F_r \frac{(x_2-a)^2}{2} + C_2 \tag{17}$$

$$E J y_2 = R_A x_2^3/6 + (q_2 - q_1) \frac{x_2^4}{24} + F_r \frac{(x_2-a)^3}{6} + C_2 x_2 + D_2 \tag{18}$$

From equation (14), we determine the integral constant D_1 from the following condition: when $x_1=0;\ y_1=0$, *and* $D_1=0$. According to our condition for formulating the moment equation, we obtain that $D_1= D_2=0$ and $C_1= C_2$.

From equation (18), we determine the integral constant C_2 using the following condition: when $x_2=l;\ y_2=0$.

Substituting the value $x_2=l$ into equation (18), we find:

$$C_2 l + R_A \frac{l^3}{6} + (q_2 - q_1) \frac{l^4}{24} + F_r \frac{(l-a)^3}{6} = 0$$

$$C_2 = - R_A \frac{l^2}{6} - (q_2 - q_1) \frac{l^3}{24} - F_r \frac{(l-a)^3}{6l} \tag{19}$$

Substituting the value of C_2 from equation (19) into equation (14), we obtain:

$$E J y_1 = R_A x_1^3/6 + (q_2 - q_1) \frac{x_1^4}{24} + [- R_A \frac{l^3}{6} - (q_2 - q_1) \frac{l^3}{24} - F_r \frac{(l-a)^3}{6l}] x_1 \tag{20}$$

After performing some transformations, we obtain:

$$E J y_2 = R_A (\frac{x_1^3}{6} - \frac{l^3 x_1}{6}) + (q_2 - q_1)(\frac{x_1^4}{24} - \frac{l^3 x_1}{24}) - F_r \frac{(l-a)^3}{6l} x_1 \tag{21}$$

Substituting the value of R_A from equation (8) into equation (21), we obtain:

$$E\,J\,y_1 = [(q_1 - q_2)\frac{l}{2} - F_r\,\frac{l-a}{l}\,]\,(\,\frac{x_1^3}{6} - \frac{l^3x_1}{6}) + (q_2 - q_1)(\frac{x_1^4}{24} - \frac{l^3x_1}{24}) - F_r\,\frac{(l-a)^3}{6l}\,x_1$$

After performing the transformation, we obtain:

$$E\,J\,y_1 = (q_1 - q_2)\frac{l}{2}\,(\,\frac{x_1^3}{6} - \frac{l^3x_1}{6}) - F_r\,\frac{(l-a)}{l}\,(\,\frac{x_1^3}{6} - \frac{l^3x_1}{6}) + (q_2 - q_1)\left(\frac{x_1^4 - l^3x_1}{24}\right) - Fr\,\frac{(l-a)^3x_1}{6l}$$

$$E\,J\,y_1 = (q_1 - q_2)\,\frac{lx_1^3}{12} - (q_1 - q_2)\frac{l}{2}\,\frac{l^3x_1}{6} - F_r\,\frac{(l-a)}{l}\,\frac{x_1^3}{6} + F_r\frac{(l-a)xl^3}{6l} + (q_2 - q_1)\,\frac{x_1^4}{24} - (q_2 - q_1)\,\frac{l^3x_1}{24} - F_r\,\frac{(l-a)^3x_1}{6l}$$

$$E\,J\,y_1 = (q_1 - q_2)\,\frac{lx_1^3}{12} - (q_1 - q_2)\,\frac{l^4x_1}{12} - F_r\,[\,\frac{(l-a)x_1^3}{6l} - \frac{(l-a)x_1l^3}{6l} + \frac{(l-a)^3x_1}{6l}\,] + (q_2 - q_1)\,\frac{x_1^4}{24} - (q_2 - q_1)\,\frac{l^3x_1}{24}$$

$$E\,J\,y_1 = (q_1 - q_2)(\frac{lx_1^3}{12} - \frac{l^4x_1}{12}) - F_r\,[\,\frac{(l-a)x_1^3 + (l-a)x_1l^3 - (l-a)^3x_1}{6l}\,] + (q_2 - q_1)(\frac{x_1^4 - l^3x_1}{24}$$

$$E\,J\,y_1 = (q_1 - q_2)(\frac{lx_1^3 - l^4x_1}{12}) - F_r\,[\,\frac{(l-a)(x_1^3 + x_1l^3) - (l-a)^3x_1}{6l}\,] + (q_2 - q_1)(\frac{x_1^4 - l^3x_1}{24} \qquad (22)$$

From formula (22), it is evident that the deflection of the roller in the contact area between the roller surface and the fabric, i.e., in the section where the fabric selvage is pulled from the working zone, depends on the intensity of the distributed force, the weight of the roller itself q_1, the intensity of the selvage pulling force q_2, the roller length l, the radial force F_r generated in the gear transmission located on the roller, and the position of the gear driving the roller a [7].

Now, let's formulate the equation for the bending of the roller in the second plane A\`y\`x\` (Figure 1.b) [8, pp. 110-113].

We determine the reaction forces R_A\` and $R_B$\`.

By formulating the moment equations at points A\` and B\`, we obtain:

$$R_B\text{\`} = \frac{M}{l};$$
$$R_A\text{\`} = -\,\frac{M}{l}$$

Let us also place the origin of coordinates at the left support at point A\`. Then, for section I $0<x_1<a$, we can write:

$$EJy\text{\`\`} = R_A\,x_1 = -\,\frac{M}{l}\,x_1 \qquad (23)$$

$$EJy\text{\`} = -M\,\frac{x_1^2}{2l} + C_1 \qquad (24)$$

$$EJy = -M\,\frac{x_1^3}{6l} + C_1x_1 + D_1 \qquad (25)$$

For section II $0<x_2<l$

$$EJy\text{\`\`} = R_A\,x_2 + M = -\,\frac{Mx_2}{l} + M(x_2 - a) \qquad (26)$$

$$EJy\text{\`} = -M\,\frac{x_2^2}{2l} + M(x_2 - a) + C_2 \qquad (27)$$

$$EJy = -M\frac{x_2^3}{6l} + M\frac{(x_2-a)}{2} + C_2x_2 + D_2 \tag{28}$$

We determine the integral constants C_1, C_2, D_1, D_2.

According to the formulation of the moment equations, the integral constants are also equal, i.e., $C_1 = C_2$ and $D_1 = D_2$.

The constants of integration will also be determined from the conditions that express the deflections at the supports A` and B`being equal to zero [9].

When $x_1 = 0$; $y_1=0$ and $D_1=0$, it follows that $D_1=D_2=0$.

From the condition $x_2 = l$ and y=0, we can determine C_2.

From equation (28), we obtain:

$$-M\frac{x_2^3}{6l} + M\frac{(x_2-a)}{2} + C_2x_2 = 0$$

$$C_2^{`} = M\frac{x_2^2}{6l} - M\frac{(x_2-a)}{2x_2} \tag{29}$$

Substituting the value of C_2 from equation (29) into equation (25), we obtain:

$$EJy_1 = -M\frac{x_1^3}{6l} + (M\frac{x_2^2x_1}{6l} - M\frac{(x_2-a)x_1}{2x_2}$$

When $x_2 = l$ and y=0, we obtain:

$$EJy_1 = -M\frac{x_1^3}{6l} + M\frac{l^2x_1}{6l} - M\frac{(l-a)x_1}{2l} \tag{30}$$

The overall deflection of the roller can be determined using the following formula:

$$y_в = \sqrt{y_{Ayx}^2 + y_{\grave{A}\grave{y}\grave{x}}} \tag{31}$$

Here, y_{Ayx} *and* $y_{\grave{A}\grave{y}\grave{x}}$ are the deflections of the roller in the planes Ayx и A`y`x` respectively.

From the obtained expressions (22) and (30), it is evident that the deflection of the roller has a complex nature. It depends on the intensity and direction of the distributed loads q_1 and q_2, i.e., the weight of the roller, the tension force of the main threads, the circumferential force F_r, and the radial force arising in the gear transmission that drives the roller. It also depends on the length l, the distance between the supports of the roller, the position of the gear on the roller shaft that drives the roller, and the placement of the fabric on the roller surface of the weaving machine [10].

It is known that the design solution of the mechanism should allow for the execution of technological processes under normal conditions and the automation of technological processes, i.e., it should enable the creation of automated enterprises in the textile industry [11].

We will analyze the impact of the specified parameters on the deflection of the roller, the execution of technological processes under normal conditions, and the automation of the weaving production.

When producing different fabric assortments on weaving machines such as the STB-180, the parameters of the roller and, consequently, the weight of the roller remain constant, i.e., the intensity of the distributed load q_1 remains a constant value [12].

The intensity of the distributed load due to the action of the main threads can vary over very wide ranges. Therefore, it is advisable to study the impact of the intensity of the distributed load on the deformation of the roller in such cases.

- In the first case, the intensity of the distributed load q_1 is equal to the intensity of the distributed load due to the action of the main threads q_2, i.e., $q_1 = q_2$.
- In the second case, the intensity q_1 is less than the intensity q_2, i.e., $q_1 < q_2$.
- In the third case, the intensity q_1 is greater than the intensity q_2, i.e., $q_1 > q_2$.

When the intensities $q_1 = q_2$, in the deflection formula (22) contains only one term, which depends on the radial force driving the roller [13].

When the intensity q_1 is smaller than q_2, in the deflection formula (21), the first term has its own sign, and the value of the third term decreases. This results in the deflection being larger than in the first case [14].

When the intensity q_1 is greater than q_2, in the deflection formula (19), the third term has its own sign, while the first term has a minimal value. In this case, the deflection of the roller is larger than in the first case [15, pp. 117-119].

Conclusion.

1. A methodology has been developed for determining the deflection of any section of the roller.
2. The equation for the deflection of the roller during the production of different fabric assortments has been formulated.
3. The equation for the deflection of the roller has been solved, and expressions for its determination have been obtained.
4. It has been established that the deflection of the roller depends on the intensity of the distributed load, its own weight, the distributed load acting force, the pulling of the fabric selvage from the working zone, the location of the gear transmitting motion to the roller relative to the roller's support, and the length of the roller.
5. It has been established that the deflection of the roller depends on the location of the gear driving the motion of the roller. When the gear is positioned between the support and the working surface of the roller, behind the support, the deflection reaches its maximum value. Therefore, when designing receiving mechanisms, the gear transmitting motion to the roller should be positioned behind the support on the surface of the roller.

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